Original corres-
pondence on
the
"Non-equilibrium
Theory":
between Clarence
Lubin and me

CM.

347 Federal Bldg.
Amarillo, Texas.
Dedember 19,1934.

Dr. C. I. Lubin
College of Engineering
University of Cincinnati
Cincinnati, Ohio.

Dear Clarence:

I am coming through Cincinnati on January 5 or 6 and would like very much to see you. I want to impose on you to some extent but even if I did not want some help I would still like to see you. You have a nasty habit of being in Canada or some other place whenever I pass through the town but I am hoping that I can catch you this time.

I have been out here in this vicinity for the last 18 months and expect to return to Washington at the first of the year. I am still trying to figure our reserves of under-ground water for pumping irrigation districts and think I need some mathematical assistance on the job. I can't get much cooperation from my chief because, I believe, that as a geologist he rather believes that the use of mathematics to a geologist ceases with arithmatic. I'm stuck in partial derivatives and so far haven't been able to root up enough talent in these great open spaces to get me out.

The flow of ground water has many analogies to the flow of heat by conduction. We have exact analogies in ground water theory for thermal gradient, thermal conductivity, and specific heat. I think a close approach to the solution of some of our problems are probably already worked out in the theory of heat conduction. Is this problem ~~////////~~ in radial flow worked out?: Given a plate of given constant thickness and with constant thermal characteristics at a uniform initial temperature to compute the temperatures thruout the plate at any time after the introduction of a sink kept at 0 temperature? And a more valuable one from our standpoint: Given the same plate under the same conditions to compute the temperatures after the introduction of a sink into which heat flows at a uniform rate? I forgot to say that the plate may be considered to have infitie areal extent.

I'm going to ask you these two questions if I get to see you but I wont expect answers unless they can be taken out of your sleeve. I've imposed enough on you in the past. I think however that the solution of these problems is of very great significance to some of the work we are called upon to do at present, such as telling the relief agencies the prospects for rehabilitation of a number of families in areas where pump irrigation is superficially feasible.

With best regards to you and your family and best wishes for pleasant holidays,

Sincerely,

Chas. V. Theis

Cincinnati, O.
Jan. 6, 1935.

Dear C. V.:

I think this is the material you want. If you have any difficulties with what I've enclosed let me know. Almost all of it came from Carslaw's "Conduction of Heat" so reference to that book would probably clear up most questions. When I saw you ~~at~~ I think I said I would send you a second letter with more details, but I believe the material enclosed will be sufficient.

Please remember me to your wife.

Yours
Clarence.

— plane

no sink $\quad$ when $t=0$ $\quad v = \ell$

then for any t $\quad v(x,y) = \ell.$

---

no sink $\quad$ when $t=0$ $\quad v = \alpha x + \beta$

$$v(x,y) = \frac{1}{4\pi k t}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(\alpha x' + \beta)\, e^{-\frac{(x-x')^2+(y-y')^2}{4kt}}\,dx'dy'$$

$$= \frac{1}{4\pi k t}\int_{-\infty}^{\infty}(\alpha x'+\beta)\,e^{-\frac{(x-x')^2}{4kt}}\left\{\int_{-\infty}^{\infty} e^{-\frac{(y-y')^2}{4kt}}dy'\right\}dx'$$

$$\int_{-\infty}^{\infty} e^{-\frac{(y-y')^2}{4kt}}\,dy'$$

put $u = \frac{(y-y')}{\sqrt{4kt}}$

$$\to -\int_{\infty}^{-\infty} e^{-u^2}\sqrt{4kt}\,du$$

$$\sqrt{4kt}\int_{-\infty}^{\infty} e^{-u^2}du = \sqrt{4\pi k t}$$

$$= \frac{1}{4\pi k t}\int_{-\infty}^{\infty}(\alpha x+\beta)\quad e^{-\frac{(x-x')^2}{4kt}}dx'\ \sqrt{4\pi k t}$$

$$\therefore v(x,y) = \frac{1}{\sqrt{4\pi k t}}\int_{-\infty}^{\infty}(\alpha x'+\beta)\,e^{-\frac{(x-x')^2}{4kt}}dx'$$

$$= \beta + \frac{\alpha}{\sqrt{4\pi k t}}\int_{-\infty}^{\infty} x' e^{-\frac{(x-x')^2}{4kt}}dx'$$

$$= \beta + \frac{\alpha}{\sqrt{4\pi k t}}\int_{-\infty}^{\infty}(x + 2\sqrt{kt}\,u)\, e^{-u^2}du\ 2\sqrt{kt}$$

$$= \beta + \frac{\alpha x}{\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-u^2}du$$

$$+ \frac{\alpha\, 2\sqrt{kt}}{\sqrt{\pi}}\underbrace{\int_{-\infty}^{\infty} u e^{-u^2}du}_{=0}$$

$$= \beta + \alpha x.$$

for any time t $\quad v(x,y,t) = \alpha x + \beta.$

Initial temp. 0
sink strength p

$$\frac{\partial v}{\partial t} = \kappa\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

$$v = \frac{q}{4\pi\kappa t}\, e^{-\frac{x^2+y^2}{4\kappa t}}$$

Carslaw, p. 152.

put $q = \phi(t')\, dt'$

$$v(x,y,t) = \int_0^t \frac{\phi(t')\,\cancel{dt}}{4\pi\kappa(t-t')}\, e^{-\frac{x^2+y^2}{4\kappa(t-t')}}\, dt'$$

$\lambda c\rho$ = quantity of heat
λ = strength which is a temperature
rate/unit time of
see derivation of $Q (=\lambda)$ in Carslaw p 152.
C V T Jan 19, 1955

Suppose $\phi(t) = \text{constant},\ \lambda$

$$v(t) = \frac{\lambda}{4\pi k}\int_0^t \frac{e^{-\frac{x^2+y^2}{4\kappa(t-t')}}}{t-t'}\, dt'$$

put $\tau = \frac{(x^2+y^2)}{4k(t-t')}$

$4\kappa t' = \frac{x^2+y^2}{4\kappa t}$
$-dt' = \frac{x^2+y^2}{4\tau^2}$

$$v(t) = \frac{\lambda}{4\pi k}\int_{\frac{x^2+y^2}{4k t}}^{\infty} \frac{e^{-\tau}}{(t-t')}\,\frac{(x^2+y^2)}{4k}\,\frac{d\tau}{\tau^2} = \frac{\lambda}{4\pi k}\int_{\frac{x^2+y^2}{4k t}}^{\infty} \frac{e^{-\tau}\,d\tau}{\tau}$$

Eulerian constant

$$\int_x^\infty \frac{e^{-\tau}\,d\tau}{\tau} = -.577216 - \log_e|x| + x - \frac{1}{2}\frac{x^2}{2!} + \frac{1}{3}\frac{x^3}{3!} - \frac{1}{4}\frac{x^4}{4!}\cdots$$

for x small.

$$\int_x^\infty \frac{e^{-\tau}\,d\tau}{\tau} = e^{-x}\left\{\frac{1}{x} - \frac{1}{x^2} + \frac{2!}{x^3} - \frac{3!}{x^4} + \cdots\cdots + (-1)^{n-1}\frac{(n-1)!}{x^n} + R_n\right\}$$ for x large.

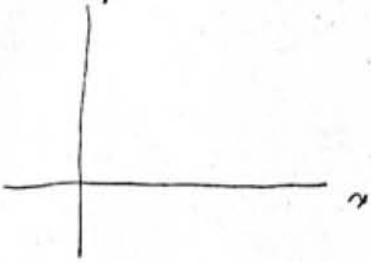
for R_n the remainder $|R_n| < n!\,x^{-(n+1)}$

Infinite series p. 334
Bromwich.

Initial temp. 0

line sink strength λ.

Carslaw p. 153



Instantaneous sink strength Q

$$v(x,y,t) = \frac{Q}{2\sqrt{\pi \kappa t}} e^{\frac{-x^2}{4\kappa t}}$$

Let $Q = \phi(t')\,dt'$

In such a sink $\quad v(x,y,t) = \int_0^t \frac{\phi(t')}{2\sqrt{\pi\kappa(t-t')}} e^{-\frac{x^2}{4\kappa(t-t')}}\,dt'$

Let $\phi(t)$ = constant λ (per unit time, for whole line)

$$v(x,y,t) = \frac{\lambda}{2\sqrt{\pi k}} \int_0^t \frac{e^{-\frac{x^2}{4\kappa(t-t')}}}{\sqrt{t-t'}}\,dt'$$

put $\quad \tau = \frac{x}{2\sqrt{\kappa(t-t')}} \qquad t-t' = \frac{x^2}{4\kappa\tau^2}$

$$-dt' = -\frac{x^2}{2\kappa\tau^3}\,d\tau$$

$$v(x,y,t) = \frac{\lambda}{2\sqrt{\pi k}} \int_{\frac{x}{2\sqrt{kt}}}^{\infty} \frac{e^{-\tau^2}}{\frac{x}{2\tau\sqrt{k}}} \frac{x^2}{2\kappa\tau^3}\,d\tau$$

Integrated by parts

$$= \frac{\lambda x}{\sqrt{\pi}\,2k} \int_{\frac{x}{2\sqrt{kt}}}^{\infty} \frac{e^{-\tau^2}}{\tau^2}\,d\tau = \frac{\lambda x}{\sqrt{\pi}\,2k} \left\{ e^{\frac{-x^2}{4kt}} \frac{2\sqrt{kt}}{x} - 2\int_{\frac{x}{2\sqrt{kt}}}^{\infty} e^{-\tau^2}\,d\tau \right\}$$

$$u = \frac{x\,(\text{feet})}{2\sqrt{\frac{T\,t\,(\text{days})}{7.48\,S}}}$$

$$\left\{ \frac{1}{\sqrt{\frac{x^2}{4\kappa t}}\, e^{\left(\frac{x^2}{4\kappa t}\right)}} - 2\int_{\sqrt{\frac{x^2}{4\kappa t}}}^{\infty} e^{-\tau^2}\,d\tau \right\}$$

$$v\,(\text{feet}) = \frac{720\,F\left(\frac{\text{gal}}{\text{min-ft}}\right)\, x\,(\text{feet})}{T\ \frac{\text{gal}}{\text{day ft}}} \left\{ \frac{e^{-u^2}}{\sqrt{\pi}\,u} - \frac{2}{\sqrt{\pi}}\int_u^{\infty} e^{-u^2}\,du \right\} \left\{ \frac{1}{u\,e^{u^2}} - 2\int_u^{\infty} e^{-u^2}\,du \right\}$$

where $u = 1.367\, x \sqrt{\frac{S}{Tt}}$

Initial temp $v = l$

Sink strength λ

$$v = l + \frac{\lambda}{4\pi k}\int_{\frac{x^2+y^2}{\cancel{4\pi k}\ 4tk}}^{\infty} \frac{e^{-\tau}}{\tau}\, d\tau$$

See p. 2.

Initial temp $v = \alpha x + \beta$.
(when $t = 0$)

sink strength q

$$v(x, y, t) = \alpha x + \beta + \frac{\lambda}{4\pi k}\int_{\frac{x^2+y^2}{\cancel{4\pi k}\ 4kt}}^{\infty} \frac{e^{-\tau}}{\tau}\, d\tau.$$

See p. 2.

Initial temperature $v = l$

line sink strength λ

$$v(x,y,t) = l + \frac{\lambda x}{\sqrt{\pi}\, 2k}\int_{\frac{x}{2\sqrt{kt}}}^{\infty} \frac{e^{-\tau^2}}{\tau^2}\, d\tau$$

See p. 3.

Initial Temperature $v = \alpha x + \beta$.

line sink strength λ.

$$v(x,y,t) = \alpha x + \beta + \frac{\lambda x}{\sqrt{\pi}\, 2k}\int_{\frac{x}{2\sqrt{kt}}}^{\infty} \frac{e^{-\tau^2}}{\tau^2}\, d\tau.$$

See p. 3.

April 23, 1935

Dr. C. I. Lubin,
College of Engineering,
University of Cincinnati,
Cincinnati, Ohio.

Dear Clarence:

I am enclosing a copy of a paper utilizing your work to be given before the Section of Hydrology of the American Geophysical Union on April 25 or 26. I regret I did not get a copy to you sooner, but it has just been finished.

There will be no time for a reply before the paper is given orally but there will be time to incorporate any criticism before it is published in the Transactions. I have been somewhat at a loss as how to handle your part in it. I should be glad to include you as one of the authors, for it could not have been written without you, but did not want to put any responsibility on you until you had a chance to criticise the mathematical form. If you wish to have your name as an author, be sure that I shall be glad to add it.

I believe that this paper is a fundamental contribution to ground water hydrology. I, personally, feel sure that your work during our visit has given me the basic theory for all my work in New Mexico and Texas. I hope we have a chance to continue the development of the theory.

I want to thank you again for the great help you gave me. Best regards to you, your family, and our common friends at Cincinnati.

Sincerely,

Chas. V. Theis
Assistant Geologist

Enclosure.

UNIVERSITY OF CINCINNATI
COLLEGE OF ENGINEERING AND COMMERCE

May 7, 1935.

Dear C. V.

I should have written you sooner but I gathered there was no hurry so my natural indolence prevailed. I read your paper and thought it very good and furthermore I consider the reference you made to me more than sufficient. I would not want to appear as co-author, first because my part in it was very small, second because from the standpoint of mathematics the work is not of fundamental importance, i.e. to mathematicians the mathematical part is not significant. I hope this does not sound snooty to you and of course it is in no sense a criticism of the paper.

In reading the paper over there were one or two minor points in the mathematical part which I believe should be changed. The most important one ~~—~~ is to use v to represent the temperature instead of the change of temperature as you indicated on page 3. Actually the temperature is equal to the change in temperature in this case because the initial temperature is zero. However I think it better to give v the meaning used in Carslaw. I have checked equations (2) (3) and (4) but have not computed the numbers appearing in (6). Also equation (7) could be obtained by using for Q in the integration leading to (2) the following

$$Q = \lambda \qquad \text{for } t = 0 \text{ to } t = t - t'$$
$$Q = 0 \qquad t = t - t' \text{ to } t = t.$$

Note this t' is not the same as the t' in equations leading to (2).

There is another thing which, however, I do not think has much significance, and that is in the heat problem for a sink λ would be negative, thus the temperature would be lowered. In your problem the pressure would be lowered (at a pt)

UNIVERSITY OF CINCINNATI
COLLEGE OF ENGINEERING AND COMMERCE

and thus be negative (?); or as I gather you have done: you have used the difference between the pressure and a fixed pressure, to correspond to the v of the heat problem. The only difference introduced this way would be a constant and a sign. I don't suppose I've made myself clear and if this is of any interest let me know and I will try to express it more clearly.

There are one or two minor suggestions; on page 2. I believe it preferable to speak of "Fourier and subsequent writers" instead of students. This is in line eleven. On page 3 it is ~~probably~~ possibly better to describe the conditions of the heat problem in more detail, i.e. in ∞ x-y plane and ~~origin~~ initial temperature zero.

In equations (3) and (4) it may cause confusion to use the letter x which has already appeared in equation (2).

at the bottom of page 4 when you introduce the terms of your problem it might be better to introduce the ~~letters~~ symbols there as well as on page 5 when they are collected.

These are the only suggestions I have. If they are not clear let me hear from you about them. As you can see they are not very important.

My classes are over now and I can spend time on a problem or so I am interested in. I only hope I shall do so there always seem so many ways of wasting time. The weather so far has been rather bad so I have not had the excuse of going out but soon I'll have that one to fall back on. I have not yet decided on my trip west. There are several people here whom I shall consult. Also I have received your information about Oregon and wish to thank you.

Please remember me to your wife.

Yours
Clarence Lubin.