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A Theory of Groundwater Motion in Small Drainage Basins in Central Alberta, Canada[1]

J. Tóth

Groundwater Division, Research Council of Alberta
Edmonton, Alberta, Canada

Abstract. On the basis of the parallel pattern of the water divides and the valleys in parts of central Alberta and the inferred difference in permeability between the Paskapoo and Edmonton formations, basins are considered to be separate units of flow in the groundwater regime. For the cross-sectional distribution of the fluid potential in a basin of homogeneous lithology an equation is found that relates the fluid potential to the acceleration of gravity, topographic gradient of the valley flank, horizontal distance between water divide and valley bottom, elevation of the water table at the valley bottom above the horizontal impermeable boundary, elevation above the horizontal impermeable boundary, and horizontal distance from the valley bottom. The validity of the assumption that groundwater runoff is discharged mainly at the valley bottoms is disputed. A boundary between areas of recharge and discharge is proved mathematically. A theory is advanced to explain the systematic deviation of the theoretical value of the fluid potential from the observed values. Local anomalies of the piezometric surface are accounted for by the presence of lenticular bodies of relatively high permeability. Mathematical formulas are used to express the relation between those anomalies and the permeability ratios and the size and shape of the lenses. A schematic cross section of possible potential distribution and flow pattern across a watershed is presented.

Introduction. An attempt is made to understand groundwater movement in small drainage basins of known physiographic and hydrogeologic characteristics. The method of reasoning employed is inductive; on the basis of field observations a mathematical model has been set up to account both for the general features of the flow systems involved and for the apparent anomalies in the general flow pattern.

The area in which the field observations were carried out is in central Alberta, Canada (Figure 1). The relatively well defined morphologic and geologic characteristics of the area offer a comparatively clear-cut basis for mathematical reasoning.

Hydrology. The area under consideration consists of a number of small drainage basins, all of which are tributary to the Red Deer River (Figure 1). The four main creeks—Lonepine, Kneehills, Threehills, and Ghostpine—run approximately parallel to one another from northwest to southeast and converge toward the southeast corner of the area. Thus, there exists a series of almost equally spaced, parallel valleys and watersheds over most of the area. The creeks meander considerably across their flat-bottomed valleys. The widths and depths of the actual water bodies are generally not more than 5 to 10 feet and are only 1 to 3 feet, respectively, during the summer. During periods of spring runoff and heavy rainfall, rushing streams develop, flooding parts of the valley bottoms. Without a considerable source of surface water, however, motion in the creeks is hardly perceptible and is in many places imperceptible. Despite this, the water in the creeks has a perennial character, which suggests a groundwater origin and thus a continuous discharge of groundwater for part of it. Dry valleys, comparable in size to the perennial creeks, exist also, however. In the dry valleys, the general form of groundwater flow must be the same as in other valleys, except that the amount of groundwater discharge is less than the potential evaporation at the bottom of the valley. It appears that even in the perennial case groundwater discharge is barely in excess of evaporation. Groundwater discharge into a stream, in amounts small relative to the size of the basin, cannot significantly affect the fluid potential distribution in the vicinity of the stream, whereas if there is a large amount of base flow the stream will act as a line sink and will have an effect on the potential distribution

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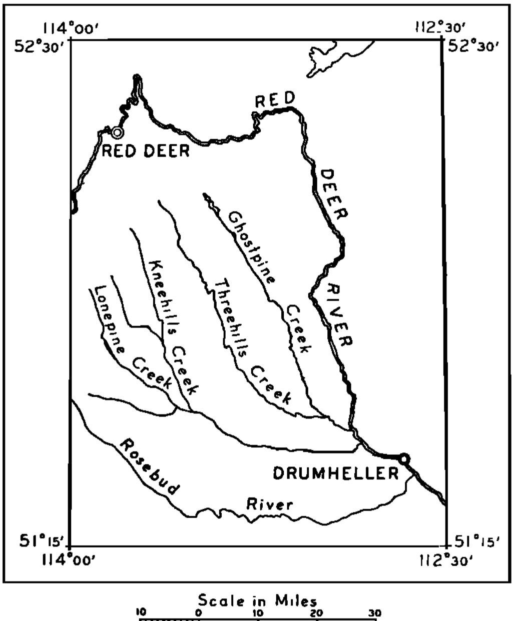

Fig. 1. Map showing location of area and parallelism of creeks.

at appreciable distances from the river. The amount of baseflow is thought to constitute one of the basic differences between a small and a large stream. As to the low rate of groundwater discharge in small creeks, reference can be found in a recent paper by *Norvatov and Popov* [1961, pp. 20–27].

With respect to the characteristics of the piezometric surface in the area, as observed in water wells and in seismic shot holes, three important features should be mentioned: (1) the close correlation of the piezometric surface with the topography in general; (2) the relatively high or low natural (i.e. no recent pumping) levels in certain wells as compared with the general piezometric surface at wells of similar depth; (3) the differing character of the change in head with changing well depth if the wells are grouped according to recharge and discharge areas. In this sense, areas located above (uphill from) the midline between a water divide and a valley bottom are referred to as areas of recharge, whereas those between the midline and the stream are considered to be discharge areas. This separation is a consequence of the theory to be developed later in this paper.

The general correlation of the piezometric surface with topography in Alberta, in wells of approximately the same depth, was described by *Farvolden* [1961, p. 13] in connection with the same rock unit (Paskapoo formation) that will be considered in the present study. *Jones* [1960, p. 21] and *Meyboom* [1961, p. 26] observed the same phenomenon in other areas of Alberta. These authors explained the correlation between the piezometric and topographic surfaces on a qualitative basis by assuming hydraulic continuity between all points of the flow system. In other words, the piezometric surface seems to be directly related to the water table. The second feature of the piezometric surface is encountered in some wells in which either higher or lower pressures prevail, producing water levels that are anomalous when compared with the general water levels in adjacent wells of the same depth. Generally, only poor or no hydraulic interrelation of such anomalous water levels can be determined, even if they have an apparent areal character. An example that may be mentioned here involves two flowing wells, about 120 feet deep, that are approximately 1000 feet apart. The drilling of a third hole, itself a flowing well, at a distance of about 1500 feet from both wells caused a serious drop of the water level in one of the wells and left the other unchanged. Therefore, whereas the anomalously high and low water levels suggest confined conditions, the local occurrence of these water levels and the possible hydrologic discontinuities between them remain unexplained. The interpretation of these phenomena by *Jones* [1960, p. 21] does not appear to be entirely satisfactory because his 'isolated aquifers that are not connected with the over-all system' does not allow for any method of recharge. With respect to the third characteristic of the piezometric surface in the surficial formations of Alberta, the observation has been made that the depths to the natural water levels in wells generally increase with increasing well depth [*Farvolden*, 1961, p. 13; *Meyboom*, 1961, p. 26–31; *Jones*, 1962]. However, only Meyboom made an attempt to find a functional relation between well depth and depth to the natural water level. On the basis of statistical analysis he calculated the regression lines for depths to static level versus well depths and found that 'there is a significant decrease in slope of regression lines, when going from a recharge area to a discharge area.' He does not indicate the possible position of the boundary between recharge and discharge areas, however. Furthermore, according to theoretical considerations, the depth to static level should decrease with increasing well depth in discharge areas. Meyboom's actual field observations show the opposite to be true, which incongruity deserves some attention. In summary, a problem arises in achieving a consistent explanation that takes into account the general close relationship of the piezometric surface to surface topography, and also the marked deviations that may take place with changes either in well depths or in well locations.

To be able to set up a mathematical model that approaches reality, we must have a good understanding of the physiographic and geologic characteristics of the area in which the observations pertinent to the establishment of the theory of groundwater flow in small basins were carried out. These characteristics will be discussed briefly.

Physiography. Physiographically, the area (Figure 1), of approximately 1800 mi², lies within the Great Central Plain region of Canada. The surface of the area slopes generally downward to the east. The average slopes are 14 ft/mi and 21 ft/mi along the north and south boundaries, respectively. The surface is very gently rolling and is subdivided by a few main creeks into nearly parallel watersheds and valleys, as was mentioned previously. The valleys are broad, with very gently sloping sides. The gradients of the valley sides vary between 40 and 100 ft/mi (from 0.008 to 0.02 radian). The distances between adjacent water divides are 6 to 10 miles, with longitudinal valley gradients of about 10 ft/mi (0.002 radian)—1/4 to 1/10 those of the sides. Because of considerable meandering of the creeks, the gradients of the actual creek beds are even less. The creeks all have tributary coulees that are dry except during periods of surface runoff. In addition to these tributary valleys, there are minor topographic highs and lows, in many cases glacial in origin. The greater part of the area is farmed, and only in the north, mainly around lakes, are there some patches of woods.

Geology. The area lies on the eastern flank of the Alberta syncline, in which sediments were deposited in Late Cretaceous and Tertiary times. This syncline originated during the uplift of the Rocky Mountains, and it underwent a slow and often interrupted subsidence during its existence as a foredeep east of the rising mountains. Only those formations are discussed below for which information is available on the groundwater regime. For this reason, the lowest stratigraphic unit considered is the Upper Cretaceous Edmonton formation.

According to *Allan and Sanderson* [1945, p. 64] the Edmonton formation consists mainly of '... poorly sorted sandstones and siltstones cemented for the greater part with bentonitic clay . . . .' The clastic constituents of the Edmonton formation are very fine-grained, containing from 8 to 35 per cent of bentonitic clay. The most important sedimentary features of the formation are foreset bedding, cross-bedded layers, lensing relationships, and isolated lenses of cross-bedded coarser sand, filling channels cut into earlier sediments. These features indicate subaerial, flood-plain deposition. The average thickness of the Edmonton formation in the area is approximately 1000 feet, and the top of the Edmonton is a subhorizontal erosion surface, dipping very gently to the west.

The Paskapoo formation of Paleocene or early Eocene age disconformably overlies the Edmonton beds. With regard to groundwater occurrence in central Alberta, this is the most important rock unit. The thickness varies from 0 to 600 or 1000 feet from east to west across the area. Despite some sedimentary similarities of the Paskapoo formation to the Edmonton formation, several distinct differences exist between the two units. According to *Allan and Sanderson* [1945, p. 27] the Paskapoo consists of soft gray, clayey sandstones, some conglomerate and soft shales, and clays. The sandstones are coarse-grained and well-sorted. The little cementing material consists mainly of carbonate. *Parks* [1916, p. 191] described the sandstones as having a 'more or less defined pepper-and-salt effect.' The amount of clay in the sandstones is consistently less than 6 per cent. Lenticular sediment bodies are typical of the Paskapoo formation. Many sandstone layers, interbedded with shale, have only a restricted areal extent and irregular bedding; pinching out of layers as well as cross bedding can be observed in all outcrops.

According to Allan and Sanderson the Paskapoo deposits are subaerial flood plain sediments. These authors suggest that the Paskapoo comprises the 'deposits of large rivers which had strong flow at their heads and which deployed upon a great, incompletely drained plain.' A problem arises in later paragraphs of this paper: little is known about the size and shape of the bodies of sandy material of relatively high permeability which are supposedly entirely surrounded by a less permeable shaly matrix. For the mathematical treatment of the potential distribution within and without these bodies, long, elongated bodies have been chosen in order to make the calculation relatively easy. Furthermore, this shape is common in the sediment types of the above formations where even the existence of shoestring sands is possible [*Pettijohn*, 1956, p. 618]. Owing to the two-dimensional treatment, the results and conclusions obtained for tubularly shaped bodies will also be applicable in first approximation to the cross sections of more-discus-like lenses. As to the sizes of these bodies, it is reasonable to assume that zones of relatively high permeabilities, several tens of feet in thickness and hundreds or even thousands of feet in length, could be distinguished. The Paskapoo formation is the surface formation of the area, but at many places it is overlain by till and glacial outwash, varying in thickness from 0 to 150 feet and being thickest in preglacial or interglacial channels.

Control of groundwater motion by topography and geology. Examination of the effects of local topography and geology on the nature of the groundwater flow and on the location and nature of the recharge in the area suggests the following conditions:

1. No confined flow systems of large areal extent can be formed.
2. Vertical impermeable boundaries can be assumed to exist for all practical purposes at water divides and streams.
3. An abrupt decrease in permeability can be considered to exist at the boundary of the Paskapoo and Edmonton formations.
4. As yet no account can be given for the effect of the glacial material on the groundwater flow system.

Four reasons can be presented in support of the first condition. First, continuous and extended layers of relatively high permeability, embedded in strata of relatively low permeability, which would enable confined systems of groundwater flow to form, are absent. However, formation of flow systems—confined and unconfined— of large areal extent is prevented also by the parallel series of river valleys, at least at elevations above which the valleys intercept the flow. Because only a very few water wells penetrate below the valley floor levels, most observations obtained from wells relate to water systems that originate and end within one interval of water divide and valley bottom. The other reasons in favor of point 1 are that the piezometric surface generally shows a close correlation with the topography and that no areal deviations from this rule are known; a piezometric surface independent of the topography should have been observed in the case of confined flow. Summarizing, it can be stated that any one watershed seems to constitute a unit system in the regime of groundwater flow, having both recharge and discharge areas existing within the boundaries of the system. In the area under consideration there are no confined flow systems of large areal extent with intake areas at outcrop, either within or without the boundaries of a unit system.

The statement in the second point is based partly on the observation that the topography is approximately symmetrical relative to either a water divide or a valley bottom, and partly on the assumption that the only recharge is infiltrating rain and melt water. Thus the distribution of the fluid potential on both sides of a divide or of a creek is also symmetrical. As a consequence, vertical planes through the axes of a watershed and of a valley bottom can be considered to play the part of vertical impermeable boundaries.

The contact between the Paskapoo and Edmonton formations is considered to be characterized by a marked change in permeability. In addition to the data on mineralogic, lithologic, and textural differences, which are sufficient to indicate a large difference in permeability of the two formations [*Todd*, 1959, p. 53], there is evidence that at many places where the Paskapoo-Edmonton contact outcrops large springs exist [*Allan and Sanderson*, 1945, p. 11]. The existence of these contact springs strongly supports the assumption of the contact being a major boundary of permeability; it is therefore treated for convenience as a horizontal impermeable boundary.

The effect of the glacial material on the groundwater flow is uncertain. The veneer of

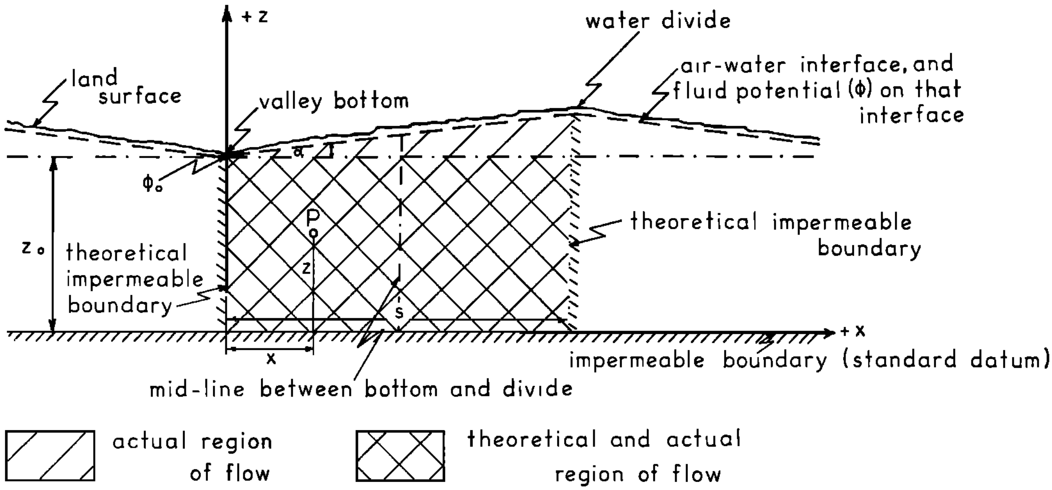

Fig. 2. Cross section of a valley, showing real and theoretical boundaries and flow regions.

till and outwash deposits is variable in thickness and in character. It may be discontinuous across even relatively small distances, and the water table is not uncommonly below the till-bedrock contact. This means that at numerous places there is no saturated flow of groundwater in the glacial materials. In the immediate vicinity of the creeks, where the till usually infills interglacial bedrock channels, the pattern of the fluid potential may be influenced by the differences in permeability. This effect is mostly of unknown significance, however, and as such has been disregarded in the present paper.

Mathematical model of groundwater flow between adjacent valley bottom and water divide. For the mathematical treatment, the sediments in a region bounded by vertical planes beneath the valley bottom and water divide and by a horizontal impermeable boundary (Figure 2) are assumed to be homogeneous and isotropic. Discussion of the effect of lenticularity follows in a later section.

For the derivations, *Hubbert*'s [1940] notation has been adopted. According to Hubbert, the fluid potential for liquids, which is a force potential, is expressed by

$$\phi = gz + \int_{p_0}^{p} \frac{dp}{\rho} \qquad (1)$$

where ϕ = fluid potential, g = acceleration of gravity, p = pressure at any point within the region of the fluid, p_0 = pressure of the atmosphere, ρ = density of liquid, and z = elevation of the point P, at which ϕ is the fluid potential, above standard datum.

Normal to the equipotential surfaces of ϕ an associated force field, $-\nabla\phi$, exists. In a homogeneous, isotropic, and porous medium a flow field is related to this force field by means of Darcy's law [*Hubbert*, 1940, p. 842].

$$\mathbf{q} = -\sigma\nabla\phi \qquad (2a)$$

or

$$\mathbf{j} = -\rho\sigma\nabla\phi \qquad (2b)$$

where $\mathbf{q}$ = vector of the total volume discharge, $\mathbf{j}$ = vector of the total mass discharge, $\sigma = k\rho/\eta$, k = coefficient of permeability, and η = viscosity of the fluid.

On account of the large difference in the surface gradient between the slope of the valley sides and of the valley floor, the problem is treated in two dimensions, assuming that the components of $\mathbf{q}$ or $\mathbf{j}$ parallel to the axis of the valley are small compared with the components perpendicular to the axis. The deeper below the surface, however, that the point is located at which $\mathbf{q}$ or $\mathbf{j}$ is investigated, the less valid is this assumption. And if the flow lines are extended sufficiently (in the case where no impermeable boundary intercepts the downward motion) they will finally merge with flow lines of deeper flow systems following the path prescribed by the regional slope of the land surface.

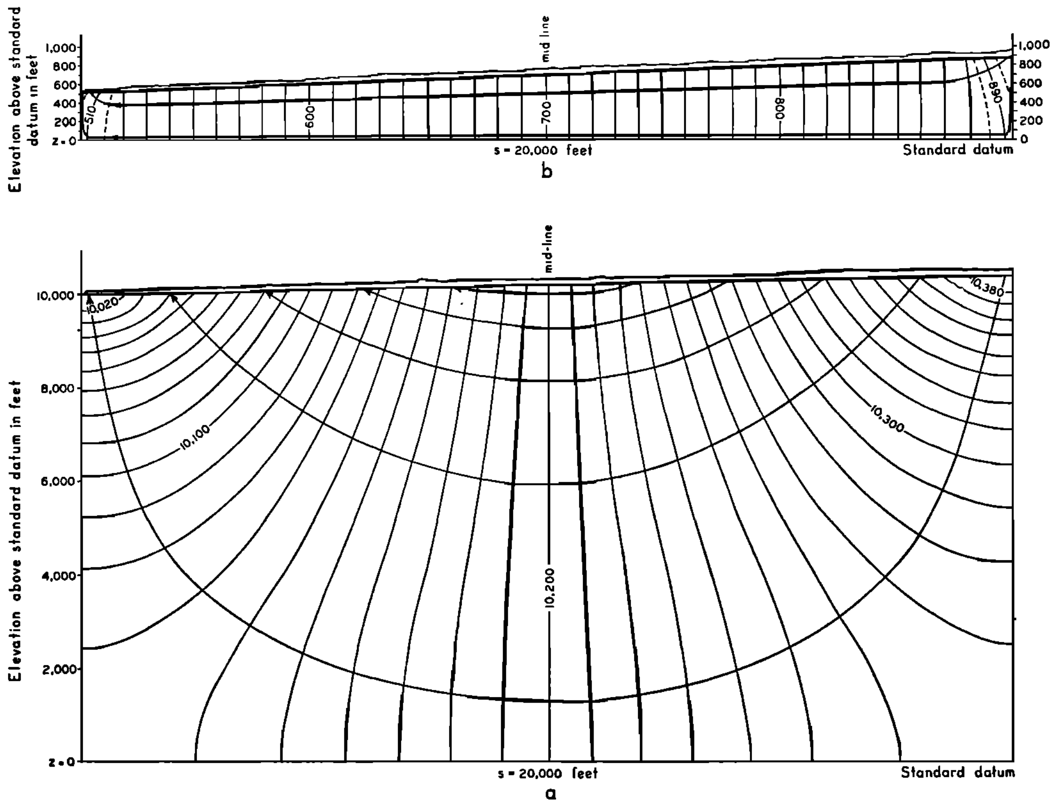

Fig. 3. Two-dimensional theoretical potential distributions and flow patterns for different depths to the horizontal impermeable boundary.

Boundary conditions. For reasons of convenience, the geometry of the region of flow has been simplified. Instead of modeling the cross section of the valley side by a trapezoid, a rectangle has been used (Figure 2). The potential on the top of this rectangle increases from the valley bottom toward the water divide at the same rate as the elevation of the surface points of the saturated zone increases. But owing to the very low angles of slope, 1° or less, a negligible distortion of the potential field will result when it is plotted and extended to the sloping surface. The mean position of the water table, the average of that of many dry and wet seasons, will closely follow the topography. In other words, the elevation z of the surface points of the water table relative to a standard datum is a linear function of the horizontal distance x of such a point from the axis of the valley. The pressure p at the water table equals the atmospheric pressure p_0; thus (1) becomes

$$\phi = gz \tag{3}$$

If the equation of the water table is $z = z_0 + cx$, where z_0 is the elevation of the water table above datum at the valley bottom, $c = \tan \alpha$, and α is the angle of slope of the water table, then the fluid potential at the water table is

$$\phi = g(z_0 + cx) \tag{4}$$

If, furthermore, g is assumed to be constant, then ϕ is a function of x only, and (4) can be written in the form

$$\phi = f(x) \tag{5}$$

Let s be the horizontal distance between the valley bottom and the water divide; then the following boundary conditions may be written for the water table and for the three impermeable

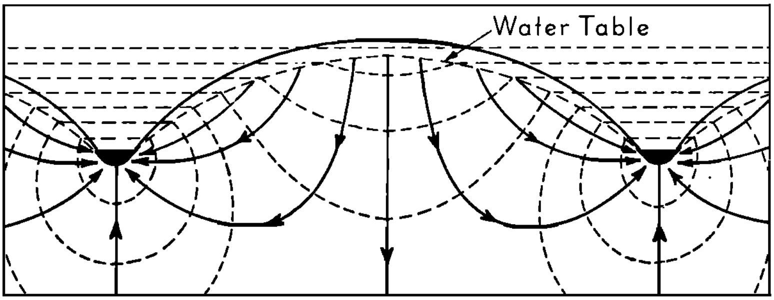

Fig. 4. Approximate flow pattern in uniformly permeable material between the sources distributed over the air-water interface and the valley sinks. (After *Hubbert* [1940].)

boundaries:

$$\phi = g(z_0 + cx) \quad \text{at } z = z_0, \quad \text{for } 0 \leq x \leq s$$

$$\partial\phi/\partial x = 0 \quad \text{at } x = 0 \text{ and } s, \quad \text{for } 0 \leq z \leq z_0$$

$$\partial\phi/\partial z = 0 \quad \text{at } z = 0, \quad \text{for } 0 \leq x \leq s$$

No point sink has been considered at the bottom of the valley because the very small or even nonexistent discharge of groundwater into the creeks or dry valleys probably hardly exceeds the rate of discharge by evapotranspiration over the whole area.

In proceeding to find the general distribution of the fluid potential, we disregard the initial period during which the region is becoming saturated, as well as the later transient periods of recharge and discharge during wet and dry seasons. Thus the basic equation for finding the potential field for this steady-state motion, in the given two-dimensional region, is the Laplace equation

$$\partial^2\phi/\partial x^2 + \partial^2\phi/\partial z^2 = 0 \qquad (6)$$

The general solution of (6) is found by separation of the variables [*Pipes*, 1958, p. 473]:

$$\phi = e^{-kz}(A \cos kx + B \sin kx) + e^{kz}(M \cos kx + N \sin kx) \qquad (7)$$

where A, B, M, and N are arbitrary constants to be found from the boundary conditions. It is found that $B = N = 0$ and $A = M$. On further development of (7) a Fourier series is obtained for ϕ:

$$\phi = \sum_{m=0}^{\infty} C_m \cos \frac{m\pi x}{s} \cosh \frac{m\pi z}{s} \qquad (8)$$

where $C_m = 2A_m$ and $m = 0, 1, 2, 3, \cdots$ C_m again can be determined from the boundary conditions, with which value the potential is obtained:

$$\phi = \frac{1}{s}\int_0^s f(x)\, dx + 2\sum_{m=1}^{\infty} \frac{\cos(m\pi x/s)\cosh(m\pi z/s)}{s\cosh(m\pi z_0/s)} \times \int_0^s f(x) \cos\frac{m\pi x}{s}\, dx \qquad (9)$$

On carrying out the integrations and substituting back into (9), the final form of ϕ is obtained:

$$\phi = g\left(z_0 + \frac{cs}{2}\right) - \frac{4gcs}{\pi^2} \cdot \sum_{m=0}^{\infty} \frac{\cos[(2m+1)\pi x/s]\cosh[(2m+1)\pi z/s]}{(2m+1)^2\cosh[(2m+1)\pi z_0/s]} \qquad (10)$$

Since all four boundary conditions are satisfied by (10), this expression of ϕ is correct for the distribution of the fluid potential in a region confined by three impermeable boundaries and by a water table with a potential which increases linearly outward from the bottom of the valley.

Illustration of the potential distribution and of the flow pattern. Numerical solutions of (10), with various parameters, were made on a digital computer. Figure 3 shows the results. The horizontal distance s between the valley bottom and water divide is 20,000 feet. The depth to the impermeable boundary, however, is 10,000 feet in Figure 3*a* and 500 feet in Figure 3*b*. In Figure 3*a*, where z_0 = 10,000 feet, the general characteristics of the potential and flow pattern are

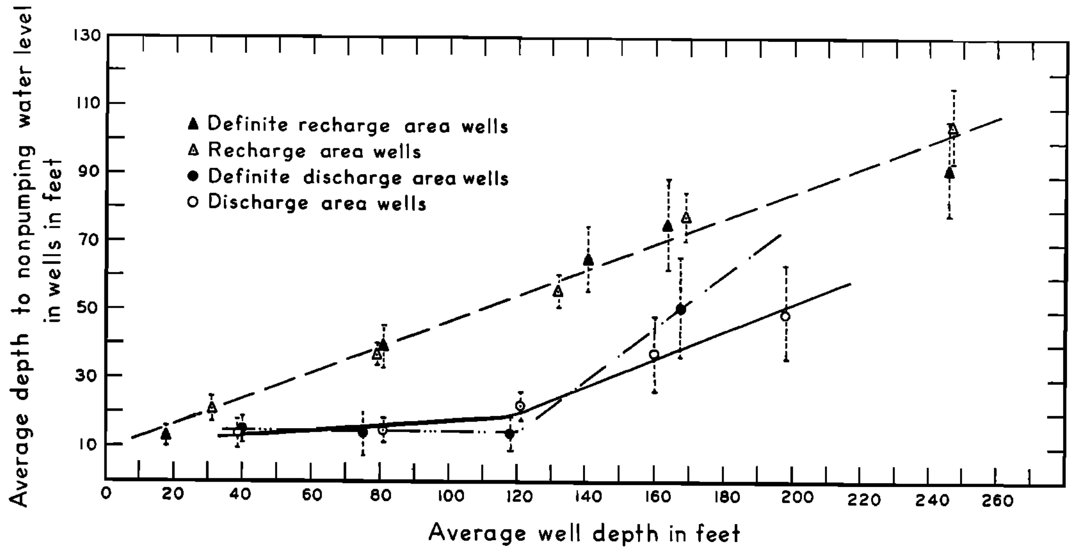

Fig. 5. Diagram showing the relation between the mean depths to water and the well depths for recharge and discharge areas.

more obvious owing to the large vertical distance. At once it can be seen that the potential distribution is symmetrical with respect to the mid-line between the valley bottom and the water divide. A flow line that originates at the air-water interface at a specific distance uphill from the mid-line will end at the same distance downhill from the mid-line.

Thus, the discharge is distributed over the area between the mid-line and the valley bottom, provided that no concentrated discharge of water takes place at the bottom of the valley or, in other words, provided that there is only, at most, a minor stream running through the valley. If, on the other hand, there is a stream that discharges an appreciable proportion of the groundwater flow, a composite discharge pattern may result, a combination of the type treated here and that presented by *Hubbert* [1940, p. 928] for which 'sinks are limited to the bottoms of valleys containing streams.' It is believed that Hubbert has overemphasized the importance, for the determination of the over-all flow pattern, of the angles between the flow lines and the air-water interface [*Hubbert*, 1940, p. 926–930]. In Hubbert's model (Figure 4), therefore, only those flow tubes that are adjacent to the stream follow rectilinear paths, whereas, according to the present considerations, this situation occurs in the vicinity of the mid-line of the valley flank. In Hubbert's figure, however, the vertical scale is considerably exaggerated; if both scales were the same, a straight and almost vertical equipotential line would be obtained proceeding downward from about the mid-point of the valley slope. This line and its vicinity are considered to be the sites of the most rectilinear flow paths, instead of the immediately adjacent areas to the creek. In Figure 3*a* the angles between the flow lines and the air-water interface gradually decrease from 90° at the top to 0° at the mid-line and increase again from 0° to 90° towards the valley bottom, with the flow vectors pointing downward in the upper half of the valley flank and toward the surface in the lower half. Concavity of the flow lines results. Flow is most intense in the vicinity of the divide and at the bottom of the valley, and the intensity of the flow decreases with depth. An equipotential line approaching the surface from the saturated zone will cross the upper boundary of that zone without interruption and will be refracted horizontally by the air-water interface, as was observed by *Hubbert* [1940, p. 928]. The negative gradient of these horizontal isobaric surfaces—assisted by capillarity—is the driving force that brings about the downward infiltration of the recharging rain water to the air-water interface. The same applies to all flow systems of similar boundary conditions. However, the shallower the depth to the impermeable boundary at the bottom of the system, the less arcuate are the

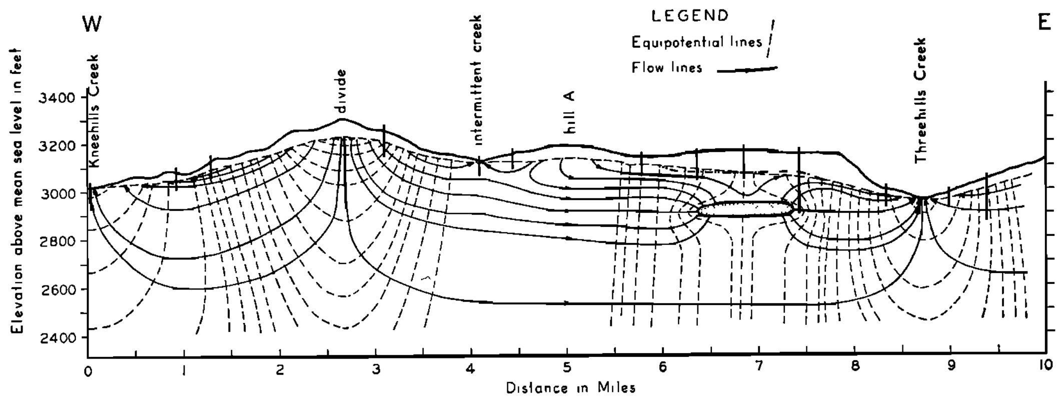

Fig. 6. Diagram showing approximate potential distribution and flow pattern, a local flow system, and the theoretical effect of a highly permeable body on the flow system across two adjacent valley sides.

flow paths. Thus, in Figure 3*b*, where $z_0 = 500$ feet, the flow is rectilinear and nearly horizontal throughout, except in the deepest and highest parts of the valley.

The phreatic fluctuation. An interesting corollary should be noted on the basis of the foregoing discussion, with respect to the type of the fluctuation of the water table. As the saturated flow of groundwater takes place from the upper half of a valley flank toward the lower half of it, a hinge-like seasonal fluctuation of the water table must result, the hinge point being located halfway between divide and stream. Thus, if other components of the decline and of the rise of the water table can be disregarded, such as those due to pumping, evaporation, vegetal transpiration, and precipitation, the largest decline will occur at the crest, whereas the elevation of the water table at the hinge will be constant. Such a situation will prevail during the winter when, because of freezing of the ground, the effects of evapotranspiration and precipitation are effectively nil. The position of the largest theoretical rise is at the creek, but since the creek plays the part of a water-table regulator, a maximum rise is not achieved. Therefore, since the two relatively stable points are at the valley bottom and at the mid-line, and because there are other components of recharge and discharge, the phreatic belt must be wider at the upper half of the valley flank than between the mid-line and valley bottom.

Variation of the fluid potential with depth. Figures 3*a* and 3*b* show a very important property of the theoretical potential distribution in a homogeneous and isotropic medium having the given boundary conditions: the decreasing fluid potential with depth in that part of the flow system on the divide side of the mid-line and the increasing potential with depth in the other half. This is detectable in the theoretical case only where the ratio of the depth of the impermeable boundary to the horizontal distance between bottom and crest of the valley is sufficiently large not to have straight equipotential lines in the middle part of the region (Figure 3*a*). In reality, however, the increase in head, as required by the theory for the lower half of the valley flank, may be observed only at the deepest parts of the valley.

In an attempt to obtain a closer approximation of the relation between the average depth to water in wells and well depth, a diagram was constructed (Figure 5). A separation was made between wells in recharge areas and those in discharge areas, the theoretical boundary between these areas being the mid-line between water divide and valley bottom. Approximately 200 wells from the area under consideration have been grouped into five classes, each class encompassing a 50-foot interval on the basis of well depth. The mean depth to water, with one standard error of the mean [*Moroney*, 1960, p. 61] for the wells of each group, is plotted against the mean depths of the well groups. Furthermore, in selecting these groups of wells a distinction has been made between those wells located in definite discharge areas (immediate vicinity of the creeks) and those located in definite recharge areas (immediate vicinity of the divides). By doing so, a means is obtained for deciding whether or not the hydrologic characteristics of wells located in recharge and discharge areas as defined by the mid-line theory are similar to those of known locations. The curve showing the relation between depth to water and well depth in wells in the definite discharge area is approximately a horizontal line to about 120 feet in well depth, beyond which the slope of the line increases sharply. Also, the mean water levels in wells located in discharge areas in general show a considerable lowering of water level with increasing well depth after an initial, almost constant, water level. On the other hand, there is no marked change in the slope of the line showing the mean water levels measured in 'definite recharge areas' and in 'recharge areas in general.' Both curves (only one is plotted) indicate a steady drop in water level with depth through all well groups. This drop, however, is at an apparently higher rate than tbat expected on the basis of the theoretical results shown in Figure 3. Moreover, the same phenomenon can be observed in wells located close to the mid-line and which thus should have no pronounced change in water level with depth. The following explanation accounts for the discrepancy between theory and observation.

Owing to the sedimentary structure of the formations, the region consists geologically of more and less permeable zones. In this environment the waters contained by beds of relatively high permeability can be looked upon as being semiperched water bodies. A decrease in potential with depth will result [*Meinzer*, 1923, p. 41]: a deviation from the fluid potential in a homogeneous medium which is expressed by (1). This same reasoning is valid for all parts of the flow system, except for the shallow depths, where, owing to local topographic irregularities (Figure 6), small, local flow systems occur which, because of their shallow depth of penetration, are largely uninfluenced by nonhomogeneities; thus the increase of head with depth in discharge areas can counterbalance the decrease in head caused by the nonhomogeneties. Below the local flow systems, however, the recharge or the mid-line areas of larger systems probably occur which do not have the pronounced increase-in-head-with-depth characteristic, and thus the decrease in head caused by the nonhomogeneities freely prevails. The above method of well-data analysis is considered to provide a means of separating different flow systems in a vertical sense.

In this regard, it should be mentioned that the flow system originating at hill A (Figure 6) is a local system relative to the 'Divide–Threehills Creek' system and is superimposed on it. The depth below which the potential commences to drop in discharge areas is between 100 and 150 feet in this part of central Alberta, as indicated by Figure 5. This depth is considered to be the average depth of the local flow systems, if the above-mentioned flow system is regarded as a regional system.

Lenses. In this section an attempt is made to determine the distorting effect of the lenses of high permeability on the original homogeneous field of fluid potential. For the shape of the model lenses, long ellipsoids have been selected. As was mentioned in the section on geology, these long, tubular bodies probably do exist in the Paskapoo type of sediments; moreover their mathematical treatment is easy, and the results also give a good approximation in two dimensions for more discus-like lenses.

The analogy for a lens of different permeability to that of the surrounding matrix, placed in a steady flow system, which is the case considered in the general derivation, has been obtained from the study of heat flow [*Carslaw and Jaeger*, 1959].

For an ellipsoid of permeability k' in a medium of permeability k, it is found that the potential ϕ_i inside and ϕ_o outside the ellipsoid are [*Carslaw and Jaeger*, 1959, p. 427]

$$\phi_i = \frac{x\,\partial\phi/\partial x}{1 + A_0(\epsilon - 1)} + \frac{y\,\partial\phi/\partial y}{1 + B_0(\epsilon - 1)} + \frac{z\,\partial\phi/\partial z}{1 + C_0(\epsilon - 1)} \qquad (11)$$

$$\phi_0 = x\frac{\partial\phi}{\partial x} + y\frac{\partial\phi}{\partial y} + z\frac{\partial\phi}{\partial z} - \frac{(\epsilon - 1)A_\lambda x\,\partial\phi/\partial x}{1 + A_0(\epsilon - 1)} - \frac{(\epsilon - 1)B_\lambda y\,\partial\phi/\partial y}{1 + B_0(\epsilon - 1)} - \frac{(\epsilon - 1)C_\lambda z\,\partial\phi/\partial z}{1 + C_0(\epsilon - 1)} \qquad (12)$$

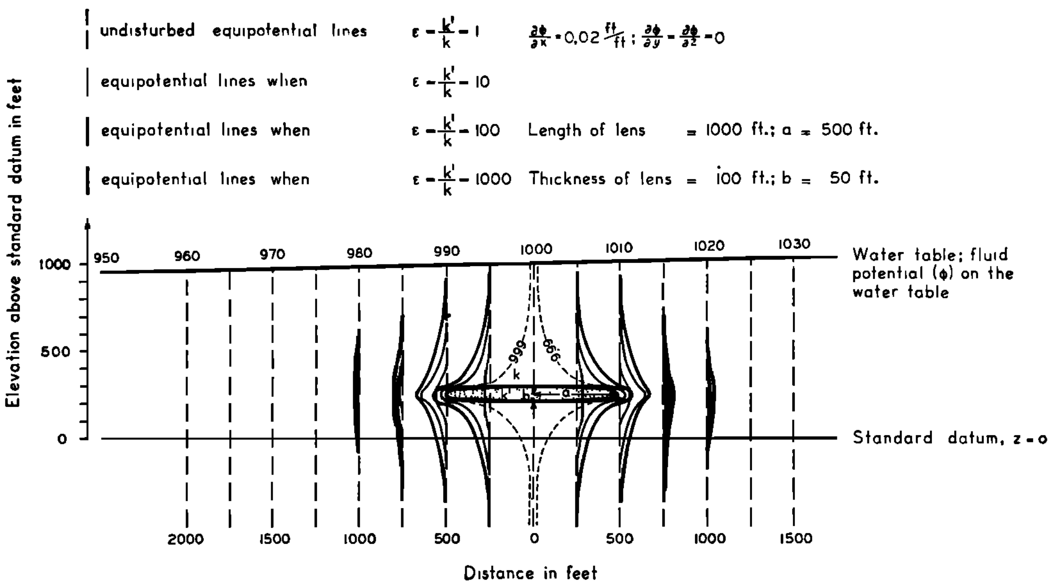

Fig. 7. Computed potential distributions around an ellipsoid of permeability k', placed in an infinite homogeneous medium of permeability k.

where ϕ is the original, undisturbed potential, $\epsilon = k'/k$, and A_0, B_0, C_0 are A_λ, B_λ, C_λ with $\lambda = 0$.

A_λ, B_λ, and C_λ are integrals [*Webster*, 1897] which, when solved for a prolate spheroid, where $b = c < a$, are expressed by

$$A_\lambda = \frac{1 - e^2}{e^3}\left(\frac{1}{2}\ln\frac{1 + e'}{1 - e'} - e'\right) \qquad (13)$$

and

$$B_\lambda = C_\lambda = \frac{1 - e^2}{2e^3} \cdot\left(\frac{e'}{1 - e'^2} - \frac{1}{2}\ln\frac{1 + e'}{1 - e'}\right) \qquad (14)$$

where $e' = [(a^2 - b^2)/(a^2 + \lambda)]^{1/2}$ is the eccentricity of the confocal ellipse through the external point considered, and $e' = e$ for $\lambda = 0$, which is the eccentricity of the generating ellipse.

For the computation of the disturbance of the fluid potential caused by the presence of a spheroidal body in a homogeneous, infinite, and steady flow it has been assumed that $\partial\phi/\partial x = 0.02$ radian and that $\partial\phi/\partial y = \partial\phi/\partial z = 0$.

The value of 0.02 radian is a typical slope of the valley walls in the area discussed. If the geologic formations in the area were homogeneous, the gradient of fluid potential in the vicinity of the mid-line between the watershed and the valley bottom would be that of the land surface. The length of the spheroid is assumed to be 1000 feet, and the diameter of the circular cross section of it is 100 feet. Computations have been made for three different ratios of permeability, namely for $\epsilon = 10$, $\epsilon = 100$, and $\epsilon = 1000$, all of which are very common in clastic sediments [*Todd*, 1959, p. 53]. The results are plotted in Figure 7. It can clearly be seen that the potential distribution inside and around the lens is symmetric relative to the axes. Denoting either $\phi_i - \phi$, or $\phi_e - \phi$ by $\Delta\phi$, the difference between the original undisturbed potential field and the new one, it is seen that $\Delta\phi$ is negative in and around the 'upstream' part and positive in and around the 'downstream' part of the lens. Its absolute value increases toward the ends and decreases beyond them, but its maximum value cannot be greater than $a\ \partial\phi/\partial x$, where a is half of the long axis of the ellipsoid and $\partial\phi/\partial x$ is the gradient of the original field of potential in the direction of a. In Figure 8 the absolute value of $\Delta\phi$ is illustrated for the end points of model ellipsoids having different thicknesses and permeability ratios. From Figure 8, it is obvious that $|\Delta\phi|$ can be increased by increasing either ϵ, a, or b. According to (11) and (12) $|\Delta\phi|$ depends also on $\partial\phi/\partial x$, which is directly related to the

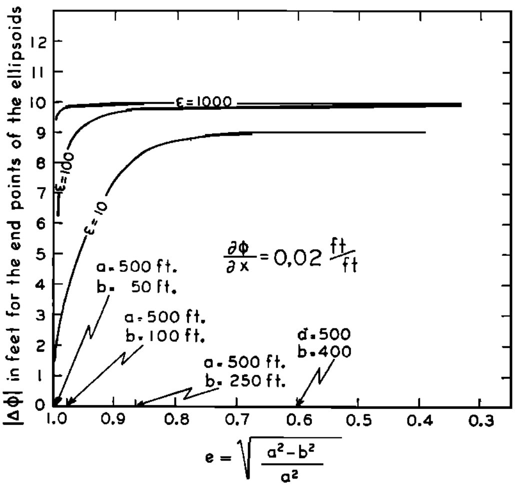

Fig. 8. Computed anomalies, in an otherwise homogeneous infinite potential field, at the end-points of ellipsoidal bodies of varying properties.

slope of the topography, as is seen from (10). Thus, the steeper the valley sides the larger the irregularities of the piezometric surface caused by lenticularity may be expected to be. It is considered that most of the anomalously high and low pressures observed in water wells in this part of central Alberta are caused by similar systems. The existence of the necessary structure is permitted by the geologic conditions, and the presence of bodies of relatively high permeability makes it possible to obtain anomalous pressure conditions of very local character. In actuality the problem is much more involved, for the sedimentary structure does not consist of a single highly permeable body embedded in a homogeneous matrix of lower permeability. Numerous bodies of high permeability are located at random relative to each other, thus possibly precluding the occurrence of a homogeneous potential field. Moreover, positive and negative values of $\Delta\phi$ may cancel or strengthen one another.

In Figure 6 the dashed line on top of an assumed sand lens of high permeability represents the observed, regular piezometric surface on the basis of water wells 100 to 150 feet deep. In one well, however, which is between 200 and 250 feet deep, the natural level is approximately 35 feet higher than that of the surrounding wells. The anomaly is explained by the presence of a highly permeable zone, illustrated schematically in the diagram. The dashed line represents the piezometric surface as it is assumed to be in wells 200 to 250 feet deep at that location.

Conclusions. On examining the computed flow systems, the following conclusions can be arrived at:

1. The piezometric surface observed in wells of approximately the same depth generally follows the topography because the flow system is essentially unconfined.

2. The flow system is symmetrical relative to the mid-line between a valley bottom and a water divide. This results in the downslope half of the valley side being the discharge area proper of the groundwater flow, and groundwater discharge is not concentrated in the valley bottom.

3. If the lithology is nonhomogeneous, and if no extensive boundary of markedly low permeability is within several hundreds of feet of the surface, the head will decrease with depth. Thus, piezometric surfaces should be drawn only on the basis of data obtained from wells of approximately equal depths.

4. Minor topographic irregularities may have their own associated flow systems, and for this reason several flow systems may be superimposed on one another. Thus, on the basis of this concept, an explanation is found for the sudden change in slope of the water level–well depth curve (Figure 5) for discharge areas between 100 and 150 feet.

5. The effects on the potential distribution of zones that differ in permeability from the matrix material are represented by anomalously low or high pressures, which are observed locally. Such high-pressure zones may give rise to flowing wells, even in areas where the general elevation of the piezometric surface is below that of the topographic surface.

6. As a consequence of the theoretical distribution of the discharge and recharge areas of the groundwater flow, the belt of the phreatic fluctuation must widen across a valley flank from the bottom to the divide.

7. The combination of (10) with (11) and (12) makes it possible to compute the effect, at any given point of the flow system, of ellipsoidal bodies entirely surrounded by a matrix of lower permeability on the distribution of the fluid potential.

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